Period 1, Oct 18, 2024

$y=(5x^4-5x^3+6x)^3 \Rightarrow y=u^3$

$u=5x^4-5x^3+6x$ $\qquad \frac{dy}{du}=3u^2$

$\frac{du}{dx}=20x^3-15x^2+6$

$\frac{du}{dx}\cdot\frac{dy}{du}=\frac{dy}{dx}$

$(20x^3-15x^2+6)(3u^2)$

$(20x^3-15x^2+6)\cdot 3(5x^4-5x^3+6x)^2$

$3(5x^4-5x^3+6x)^2 \neq (15x^4-15x^3+18x)^2$

$3(x+y)^2=3(x^2+2xy+y^2)=3x^2+6xy+3y^2 \neq$

$(3x+3y)^2=(3x+3y)(3x+3y)=9x^2+9xy+9xy+9y^2=9x^2+18xy+9y^2$

$y=(7x^3-5x^2+6x)^8 \Rightarrow y=u^8$

$u=7x^3-5x^2+6x$ $\qquad \frac{dy}{du}=8u^7$

$\frac{du}{dx}=21x^2-10x+6$

$\frac{du}{dx}\cdot\frac{dy}{du}=\frac{dy}{dx}$

$(21x^2-10x+6)(8u^7)=(21x^2-10x+6)\cdot 8(7x^3-5x^2+6x)^7$

$=8(21x^2-10x+6)(7x^3-5x^2+6x)^7$

$y=(2x^3-5x^2+6x)^5 \Rightarrow y=u^5$

$u=2x^3-5x^2+6x$

$\frac{du}{dx}=6x^2-10x+6$ $\qquad$ $\frac{dy}{du}=5u^4$

$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}}=\frac{dy}{dx}$

is correct

$(6x^2-10x+6)(5u^4)=5(6x^2-10x+6)(2x^3-5x^2+6x)^4$

$\neq$

$(6x^2-10x+6)(10x^3-25x^2+30x)^4$ is wrong

$y=\sin^4(3x^2+6x+4) \Rightarrow y=\sin^4 u \Rightarrow y=L^4$

$u=3x^2+6x+4$ $\qquad$ $L=\sin u$

$\frac{du}{dx}=6x+6$ $\qquad$ $\frac{dL}{du}=\cos u$ $\qquad$ $\frac{dy}{dL}=4L^3$

$\frac{\cancel{du}}{dx}\cdot\frac{\cancel{dL}}{\cancel{du}}\cdot\frac{dy}{\cancel{dL}}$

$(6x+6)(\cos u)(4L^3)=(6x+6)(\cos(3x^2+6x+4))(4\sin^3(3x^2+6x+4))$

wrong $\rightarrow$ $4(6x+6)(\cos(3x^2+6x+4))(\sin(3x^2+6x+4)^3)$

correct $=4(6x+6)(\cos(3x^2+6x+4))((\sin(3x^2+6x+4))^3$

$y=\sin^7(x^3+6x^2+4) \Rightarrow y=\sin^7 u \Rightarrow y=L^7$

$u=x^3+6x^2+4 \qquad L=\sin u \qquad \frac{dy}{dL}=7L^6$

$\frac{du}{dx}=3x^2+12x \qquad \frac{dL}{du}=\cos u$

$$\frac{\cancel{du}}{dx}\cdot\frac{\cancel{dL}}{\cancel{du}}\cdot\frac{dy}{\cancel{dL}}=\frac{dy}{dx}$$

$$(3x^2+12x)(\cos u)(7L^6)=7(3x^2+12x)[\cos(x^3+6x^2+4)][\sin^6(x^3+6x^2+4)]$$

$$=7(3x^2+12x)[\cos(x^3+6x^2+4)][\sin(x^3+6x^2+4)]^6$$

wrong $=7(3x^2+12x)[\cos(x^3+6x^2+4)][\sin(x^3+6x^2+4)^6]$

$y=\sin^5(x^5+6x^3+4) \Rightarrow y=\sin^5 u \Rightarrow y=L^5$

$u=x^5+6x^3+4 \qquad L=\sin u \qquad \frac{dy}{dL}=5L^4$

$\frac{du}{dx}=5x^4+18x^2 \qquad \frac{dL}{du}=\cos u$

$$\frac{\cancel{du}}{dx}\cdot\frac{\cancel{dL}}{\cancel{du}}\cdot\frac{dy}{\cancel{dL}}=\frac{dL}{dx}$$

$$(5x^4+18x^2)(\cos u)(5L^4)=5(5x^4+18x^2)[\cos(x^5+6x^3+4)][\sin^4(x^5+6x^3+4)]$$

$$=5(5x^4+18x^2)[\cos(x^5+6x^3+4)][\sin(x^5+6x^3+4)]^4$$

wrong $=5(5x^4+18x^2)[\cos(x^5+6x^3+4)][\sin(x^5+6x^3+4)^4]$

Homework

#3~~7~~, 83, 107, ~~3~~, ~~4~~, 105, ~~75~~, ~~41~~, ~~43~~, ~~69, 71, 77~~, 109, 110, ~~23, 29, 79~~

3. $F(x) = \dfrac{x+1}{x-1}$

$F(x+h) = \dfrac{(x+h)+1}{(x+h)-1}$

$$\lim_{h \to 0} \frac{\dfrac{(x+h+1)(x-1)}{(x+h-1)(x-1)} - \dfrac{(x+1)(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$\lim_{h \to 0} \frac{\dfrac{x^2 - x + hx - h + x - 1}{(x+h-1)(x-1)} - \dfrac{x^2 + hx - x + x + h - 1}{(x+h-1)(x-1)}}{h}$$

$$\lim_{h \to 0} \frac{\dfrac{\cancel{x^2} \cancel{- x} + \cancel{hx} - h \cancel{+ x} \cancel{- 1} \cancel{- x^2} - h \cancel{- hx} + \cancel{x} - \cancel{x} - h \cancel{+ 1}}{(x+h-1)(x-1)}}{\frac{h}{1}}$$

$$\lim_{h \to 0} \frac{-2\cancel{h}}{(x+h-1)(x-1)} \cdot \frac{1}{\cancel{h}} \Rightarrow \frac{-2}{(x+h-1)(x-1)} = \frac{-2}{(x-1)(x-1)}$$

$$F'(x) = \frac{-2}{(x-1)^2}$$

4. $F(x) = \dfrac{6}{x}$

$F(x+h) = \dfrac{6}{x+h}$

$$\lim_{h \to 0} \frac{\dfrac{6 \cdot x}{(x+h) \cdot x} - \dfrac{6(x+h)}{x(x+h)}}{h}$$

$$\lim_{h \to 0} \frac{\dfrac{6x}{(x+h)x} - \dfrac{6x+6h}{(x+h)x}}{h}$$

$$\lim_{h \to 0} \frac{\dfrac{\cancel{6x} - \cancel{6x} - 6h}{x(x+h)}}{\frac{h}{1}} = \lim_{h \to 0} \frac{-6\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h \to 0} \frac{-6}{x(x+h)} = \frac{-6}{x(x+0)} = \frac{-6}{x^2}$$

$$h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6 \cdot x^{\frac{1}{2}} + 3x^{\frac{1}{3}}$$

$$h'(x) = 6 \cdot \frac{1}{2} x^{\frac{1}{2} - 1} + 3 \cdot \frac{1}{3} x^{\frac{1}{3} - 1} = \frac{6}{2} x^{\frac{1}{2} - \frac{2}{2}} + \frac{3}{3} x^{\frac{1}{3} - \frac{3}{3}} = 3x^{-\frac{1}{2}} + 1x^{-\frac{2}{3}} = \frac{3}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$$

29. $F(\theta) = 3\cos\theta - \frac{\sin\theta}{4} = 3\cos\theta - \frac{1}{4}\sin\theta$

$F'(\theta) = 3\cdot(-\sin\theta) - \frac{1}{4}(\cos\theta) = -3\sin\theta - \frac{1}{4}\cos\theta = -3\sin\theta - \frac{\cos\theta}{4}$

37.

$y = x - 0.02x^2$ position

$\frac{dy}{dx} = 1 - 0.02\cdot 2x' = 1 - 0.04x$

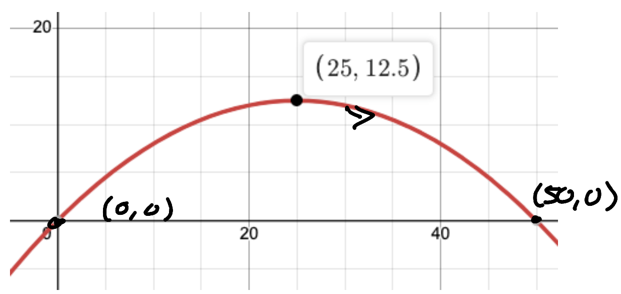
$0 = 1 - \cancel{0.04x} \Rightarrow$ highest point $\Rightarrow y = 25 - 0.02(25)^2$

$+0.04x \quad \cancel{+0.04x}$

$y = 25 - 0.02(625)$

$\frac{0.04x}{0.04} = \frac{1}{0.04} \Rightarrow x = 25$

$y = 25 - 12.5 = 12.5$



IRC

$\frac{dy}{dx} = 1 - 0.04x$

$x = 0, 10, 25, 30, 50$

x	$\frac{dy}{dx}$	
0	$1 = 1 - 0.04(0)$	going up
10	$0.6 = 1 - 0.04(10) = 1 - 0.4$	going up
25	$0 = 1 - 0.04(25) = 1 - 1$	Max height
30	$-0.2 = 1 - 0.04(30) = 1 - 1.2$	going Down
50	$-1 = 1 - 0.04(50) = 1 - 2$	going Down

distance $y = 0$

$0 = x - 0.02x^2$

$0 = x(1 - 0.02x)$

$x = 0$ or $1 - 0.02x = 0$

$x = \frac{1}{0.02} = 50$

41.

$$F(x)=(5x^2+8)(x^2-4x-6)$$

$$F'(x)=10x(x^2-4x-6)+(5x^2+8)(2x-4)$$

---

43. $h(x)=\sqrt{x}\sin x = x^{\frac{1}{2}}\cdot\sin x$

$$h'(x)=\frac{1}{2}x^{-\frac{1}{2}}\cdot\sin x + x^{\frac{1}{2}}\cdot\cos x$$

$$=\frac{1}{2\sqrt{x}}\cdot\sin x+\sqrt{x}\cos x=\frac{\sin x}{2\sqrt{x}}+\sqrt{x}\cos x$$

69, ~~71, 75, 77,~~ 79

$$h(x)=\left(\frac{x+5}{x^2+3}\right)^2 \Rightarrow y=u^2$$

$$\frac{dy}{du}=2u$$

$$u=\frac{x+5}{x^2+3}$$

$$\frac{du}{dx}=\frac{1(x^2+3)-(x+5)(2x)}{(x^2+3)^2}$$

$$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}}=\frac{dy}{dx}$$

$$\left(\frac{x^2+3-2x(x+5)}{(x^2+3)^2}\right)\cdot 2\left(\frac{x+5}{x^2+3}\right)=\frac{x^2+3-2x^2-10x}{(x^2+3)^2}\cdot 2\frac{(x+5)}{(x^2+3)}=\frac{(-x^2-10x+3)2(x+5)}{(x^2+3)^3}$$

---

71. $F(x) = (x^2-1)^{5/2}(x^3+5)$

$F'(x) = \left[\frac{d}{dx}\left[(x^2-1)^{5/2}\right]\right](x^3+5) + (x^2-1)^{5/2}(3x^2) = 5x(x^2-1)^{3/2}(x^3+5) + 3x^2(x^2-1)^{5/2}$

$y = (x^2-1)^{5/2} \Rightarrow y = u^{5/2}$

$u = x^2-1 \qquad \frac{dy}{du} = \frac{5}{2}u^{\frac{5}{2}-1} = \frac{5u^{3/2}}{2}$

$\frac{du}{dx} = 2x$

$\frac{du}{dx}\cdot\frac{dy}{du} = 2x \cdot \frac{5}{2}(x^2-1)^{\frac{3}{2}} = 5x(x^2-1)^{3/2}$

75

$y = \frac{x}{2} - \frac{\sin 2x}{4} = \frac{1}{2}x - \frac{1}{4}\cdot\sin 2x$

$\frac{dy}{dx} = \frac{1}{2} - \frac{1}{4}\left[\frac{d}{dx}(\sin 2x)\right] = \frac{1}{2} - \frac{1}{4}\cdot(2\cos 2x) = \frac{1}{2} - \frac{\cos 2x}{2} = \frac{1-\cos 2x}{2}$

$y = \sin 2x \Rightarrow y = \sin u$

$u = 2x \qquad \frac{dy}{du} = \cos u$

$\frac{du}{dx} = 2$

$\frac{du}{dx}\cdot\frac{dy}{du} = 2\cdot\cos u = 2\cos 2x$

77. $y = \frac{2}{3}\sin^{3/2} x - \frac{2}{7}\sin^{7/2} x$

$y = \sin^{7/2} x \Rightarrow y = u^{\frac{7}{2}}$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{dy}{du} = \frac{7}{2} u^{5/2}$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$\cos x \cdot \frac{7}{2} u^{5/2} = \frac{7}{2} \cos x \cdot \sin^{\frac{5}{2}} x$

$\frac{dy}{dx} = \frac{2}{3} \cdot \frac{3}{2} \cos x \cdot \sin^{\frac{1}{2}} x - \frac{2}{7} \cdot \frac{7}{2} \cos x \sin^{5/2} x$

$y = \sin^{3/2} x$

$\frac{dy}{dx} = \frac{3}{2} \cos x \cdot \sin^{\frac{1}{2}} x$

79. $y = \frac{\sin \pi x}{x+2} \Rightarrow \frac{dy}{dx} = \frac{(\pi \cos \pi x)(x+2) - (\sin \pi x)(1)}{(x+2)^2}$

$y = \sin \pi x \Rightarrow y = \sin u$

$u = \pi x$

$\frac{du}{dx} = \pi$

$\frac{dy}{du} = \cos u$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \pi \cdot \cos u = \pi \cdot \cos \pi x$

$y = \frac{1}{2} \csc 2x \quad (\frac{\pi}{4}, \frac{1}{2})$

$u = 2x$

$\frac{du}{dx} = 2$

$y = \frac{1}{2} \csc u$

$\frac{dy}{du} = \frac{1}{2} \cdot (-\csc u \cot u)$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$\cancel{2}(\cancel{\tfrac{1}{2}})(-\csc u \cot u)$

$-\csc 2x \cot 2x = -\csc \frac{\pi}{2} \cot \frac{\pi}{2}$

$= -1 \cdot \frac{0}{1} = -1 \cdot 0 = 0$

$2 \cdot \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

$\cos \frac{\pi}{2} = 0$

$\sin \frac{\pi}{2} = 1$

$\csc \frac{\pi}{2} = \frac{1}{\sin \frac{\pi}{2}} = \frac{1}{1}$

105

$\sqrt{xy} = x - 4y$

$x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} = x - 4y$

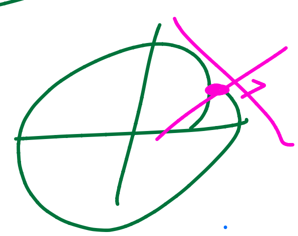
$\frac{1}{2}x^{-\frac{1}{2}} \cdot y^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot \frac{1}{2}y^{-\frac{1}{2}} \frac{dy}{dx} = 1 - 4\frac{dy}{dx}$

$x \sin y = y \cos x$

$1 \cdot \sin y + x \cdot \cos y \cdot \frac{dy}{dx} = \frac{dy}{dx}\cos x + y(-\sin x)$

$\sin y + x \cos y \frac{dy}{dx} = \cos x \frac{dy}{dx} - y \sin x$

$x^2 + y^2 = 10$ Circle



$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}y} = \frac{-x}{y}$ at $(3,1)$ what is the slope of tangent line

$\frac{-3}{1} = m = -3$ Point $(3,1)$

$y - 1 = -3(x - 3)$

Normal Line slope $= -\left(\frac{1}{-3}\right) = \frac{1}{3} = m$

$y - 1 = \frac{1}{3}(x - 3)$ Point $(3,1)$